Implementation of the Deutsch-Jozsa algorithm with Josephson charge qubits

Jens Siewert[1,2] and Rosario Fazio[2,3]

[1] *Institut für Theoretische Physik, Universität Regensburg, 93040 Regensburg, Germany*

[2] *Dipartimento di Metodologie Fisiche e Chimiche (DMFCI), Università di Catania, viale A.Doria 6, I-95125 Catania, Italy, and Istituto Nazionale per la Fisica della Materia (INFM), Unità di Catania*

[3] *National Enterprise for Nanoscience and Nanotechnology (NEST) - INFM, and Scuola Normale Superiore, I-56126 Pisa, Italy*

December 24, 2001

Abstract

We investigate the realization of a simple solid-state quantum computer by implementing the Deutsch-Jozsa algorithm in a system of Josephson charge qubits. Starting from a procedure to carry out the one-qubit Deutsch-Jozsa algorithm we show how the N-qubit version of the Bernstein-Vazirani algorithm can be realized. For the implementation of the three-qubit Deutsch-Jozsa algorithm we study in detail a setup which allows to produce entangled states.

PACS numbers: 85.25.Cp, 73.23.-b, 03.67.Lx

1 Introduction

The growing interest in quantum computation has stimulated an impressive activity in the field of 'quantum hardware'. Numerous proposals to implement quantum bits and simple one-bit and two-bit operations have appeared in many different areas of contemporary physics research. Yet the possibility to taylor a controllable two-state system is by far not enough to do quantum computation. From an engineering point of view the design of a quantum computer is difficult because of the enormous complexity of requirements, such as the possibility to prepare and to measure states easily, a highly flexible setup with a sufficiently large parameter space that can be controlled, the maintainance of coherence etc. Therefore, the touchstone of practical quantum computation is the implementation of quantum algorithms.

This goal might appear too ambitious, in particular if one thinks of realizing a set of universal gates [1], i.e. a quantum computer which can do any operation. However, in order to implement one particular quantum algorithm it is not necessary to go as far as this. Since usually one has to deal with a well-defined set of input and output states it suffices to implement just those gates which represent the desired operations on applying the gate to the possible input states. This renders the task easier than designing gates which represent the operations on application to any state.

Another advantage of the restriction to a particular algorithm is the possibility to use more complex operations (for example N-bit gates instead of a sequence of one-bit and two-bit operations) which require less computational time. This could be one way to overcome the limits which are set by a small decoherence time in a given physical system.

Surprisingly, there has been comparatively little activity towards the implementation of quantum algorithms in real physical systems. To date, quantum algorithms have been implemented by using liquid-state NMR [2, 3, 4, 5, 6, 7, 8, 9], in atomic physics [10] and by optical interferometry [11]. A solid-state implementation of Grover's algorithm has been proposed theoretically [12].

The quest for large scale integrability has stimulated an increasing interest in superconducting nanocircuits [13, 14, 15, 16, 17] as possible candidates for the implementation of a quantum computer. The recent experimental breakthrough for Josephson qubits [18, 19, 20] is the first important step towards a solid-state quantum computer.

Naturally the question arises whether, at the present technological level, it is possible to implement also quantum algorithms in these systems. Here we concentrate on charge qubits [13, 14, 15] and show how the Deutsch-Jozsa algorithm [21, 22, 23, 24] and the Bernstein-Vazirani algorithm [26] can be run on a Josephson quantum computer. We analyse the experiment by Nakamura et al. [18] in terms of quantum interferometry [23] and show that it corresponds to the implementation of the one-qubit version of Deutsch's algorithm. By generalising this idea we show how the N-qubit Deutsch algorithm, with $N \leq 3$, can be implemented.

2 Deutsch-Jozsa algorithm

Consider the subset of Boolean functions $f : \{0,1\}^N \to \{0,1\}$ with the property that f is either constant or balanced (that is, it has an equal number of 0 outputs as 1s). The Deutsch-Jozsa algorithm [21, 22, 23] determines – for a given unknown function f – whether the function is constant or balanced. With a classical algorithm, this problem would, in the worst case, require $2^{N-1}+1$

evaluations of f whereas the quantum algorithm solves it with a single evaluation by means of the following steps (here we focus on the refined version by Collins et al. [24], see also Fig. 1).

(i) All qubits are prepared in the initial state $|0\rangle$, therefore the N-qubit register is in the state $|00\ldots0\rangle$.

(ii) Perform an N-qubit Hadamard transformation $\mathcal{H}$

$$|x\rangle \xrightarrow{\mathcal{H}} \sum_{y\in\{0,1\}^N} (-1)^{x\cdot y}\, |y\rangle \ , \quad (x \in \{0,1\}^N), \tag{1}$$

where $x\cdot y = (x_1 \wedge y_1) \oplus \ldots \oplus (x_N \wedge y_N)$ is the scalar product modulo two. This is equivalent to performing a one-bit Hadamard transformation to each qubit individually.

(iii) Apply the f-controlled phase shift U_f [23, 24]

$$|x\rangle \xrightarrow{U_f} (-1)^{f(x)}\, |x\rangle \ , \quad (x \in \{0,1\}^N)\ . \tag{2}$$

(iv) Perform another Hadamard transformation $\mathcal{H}$.

(v) Measure the final state of the register. If the result is $|00\ldots0\rangle$ the function f is constant; if, however, the amplitude $a_{|00\ldots0\rangle}$ of the state $|00\ldots0\rangle$ is zero the function f is balanced. This is because

$$a_{|00\ldots0\rangle} = \frac{1}{2^N} \sum_{x\in\{0,1\}^N} (-1)^{f(x)} \ . \tag{3}$$

We note that it is reasonable to identify functions f whose outputs can be mapped into each other by a bitwise NOT. For these functions, the gate operations in Eq. (2) differ merely by a global phase factor (-1). We will use the convention $f(00\ldots0) = 0$. This reduces the number of gates which need to be implemented, by a factor of two and does not affect the exponential speed-up.

In order to implement the algorithm it is necessary to show that each individual step (preparation of the state, gate operations, measurement) can be realized in a given system. It is well known how to prepare and to measure the states in Josephson charge qubits [13, 15, 18]. Our task is to demonstrate that the gate operations corresponding to <u>all</u> admissible functions f can be performed with a single device. An important aspect of our proposal is that the gate operations are represented in a basis of superpositions of charge states.

It is interesting to note that according to Ref. [24] the Deutsch-Jozsa algorithm does not involve entanglement for $N \leq 2$, i.e. the $N = 2$ version can be realized with uncoupled qubits. On the other hand, the implementation of the algorithm for $N > 2$ involves entanglement [25] and therefore requires a setup of coupled qubits.

3 Bernstein-Vazirani algorithm

The Bernstein-Vazirani algorithm [26, 23] is analogous to the N-bit Deutsch-Jozsa algorithm described in the previous section, with the difference that the function f has the form

$$f = a \cdot x \oplus b \quad , \quad (a, x \in \{0,1\}^N,\ b \in \{0,1\}) \quad . \tag{4}$$

The gate in step *(iii)* is denoted by U_a and assumes the form

$$|x\rangle \xrightarrow{U_a} (-1)^{a\cdot x \oplus b}\, |x\rangle \quad . \tag{5}$$

By measuring the register after running the algorithm once one obtains the number a in binary representation which classically would require N function calls. The Bernstein-Vazirani algorithm does not involve entangled states at all [27]. This can be seen explicitely by rewriting the gate U_a as a product of single-qubit gates

$$U_a = (-1)^b \prod_{j=1}^{N} (\sigma_z^{(j)})^{a_j} \tag{6}$$

where a_j denotes the jth digit of a in binary representation. Here we have used the Pauli operators $\sigma_k^{(j)}$ acting on the computational basis of the jth qubit $\{|0_j\rangle, |1_j\rangle\}$ and the definition $(\sigma_k)^0 :=: I_1$ (with the one-qubit identity operator I_1). In particular we choose $\sigma_z^{(j)}\ |0_j\rangle = +|0_j\rangle$, $\sigma_z^{(j)}\ |1_j\rangle = -|1_j\rangle$.

Apart from the global phase $(-1)^b$ the set of gates U_a represents the part of the Deutsch algorithm with completely separable gates. As the algorithm starts with a product state, no entanglement is involved at any step. We note that by rewriting the action of the whole algorithm as

$$\mathcal{H} U_a \mathcal{H}\ |00\ldots0\rangle \;=\; (-1)^b \prod_j (\sigma_x^{(j)})^{a_j}\ |0_j\rangle \;=\; (-1)^b \prod_j |a_j\rangle$$

one sees that it leads trivially to the result. In conclusion, it is possible to realize the Bernstein-Vazirani algorithm using uncoupled qubits in complete analogy with the implementation for the one-qubit and two-qubit Deutsch algorithm.

4 Josephson charge qubits and implementation of algorithms without entanglement

A Josephson charge qubit [13, 15] is a Cooper-pair box (see Fig. 3a) which is characterized by two energy scales, the charging energy $E_{\rm ch} = (2e)^2/(2C)$ (here C is the total capacitance of the island) and the Josephson energy $E_J \ll E_{\rm ch}$ of the tunnel junction. At low temperatures $T \ll E_J$ only the two charges states with 0 and 1 excess Cooper pair on the island are important and the system behaves as a two-level system with the Hilbert space $\{|0\rangle, |1\rangle\}$ and the one-qubit Hamiltonian

$$H_{1\mathrm{q}} = (E_{\mathrm{ch}}/2)\ (2n_x - 1)\ \sigma_z \ -\ (E_J/2)\ \sigma_x \quad . \tag{7}$$

Here $n_x = C_g V_g/(2e)$ is the offset charge which can be controlled by the gate voltage.

In a recent experiment, Nakamura et al. have measured Rabi oscillations in a Josephson charge qubit [18]. In the following we analyse briefly the experiment and argue that it can be regarded as the one-qubit implementation of Deutsch's algorithm.

The experiment is shown schematically in Fig. 2. Let us first consider the part between the two sudden switchings of the gate voltage. The system has been prepared in a superposition of the two states $|+\rangle$, $|-\rangle$ which are defined as

$$|\pm\rangle \ = \ \frac{1}{\sqrt{2}}\ (|0\rangle \ \pm\ |1\rangle) \quad . \tag{8}$$

Now a rotation

$$\exp\left(i(E_J t/2\hbar)\sigma_z\right) \tag{9}$$

is performed on the state which results in the final state

$$\frac{1}{\sqrt{2}}(\ |+\rangle \ +\ e^{-iE_J t/\hbar}\ |-\rangle\ ) \quad .$$

Note that here σ_z is the Pauli operator with respect to the basis $\{|+\rangle, |-\rangle\}$.

After sweeping back the gate voltage, a charge measurement is performed. That is, the final superposition is measured in the charge basis $\{|0\rangle, |1\rangle\}$. In particular, for $t = 2\pi\hbar/E_J$ the outcome of an ideal measurement is $|0\rangle$ while for $t = \pi\hbar/E_J$ the state $|1\rangle$ is found.

In a spin-related language the experiment could be described as follows. The charge states form the z basis, the charging energy corresponds to the magnetic field in z direction, and the Josephson energy corresponds to the field in x direction (cf. Eq. (7)). The state is prepared with the magnetic field in z direction. Switching the gate voltage suddenly to the degeneracy point flips the magnetic field to the x direction. Thus, the spin starts to precess around the x axis. After the operation time t the field in z direction is switched on again, thus freezing the z component of the spin. The latter corresponds to the island charge which then is measured.

In order to display the analogy between Deutsch's algorithm (see Section 2) for one qubit and Nakamura's experiment, we rephrase the algorithm in the following way. In step *(i)* and *(ii)* the symmetric superposition of the states of the computational basis $\{|+\rangle, |-\rangle\}$ is prepared. On applying U_f in step *(iii)* the sign of $|-\rangle$ in the superposition is changed if the function f is balanced, or it is left unchanged. The purpose of the second Hadamard gate is the transformation of the superposition to a pure state which is to be measured.

Now compare this sequence to Nakamura's experiment. It starts with the preparation of the symmetric superposition. The Rabi oscillation corresponds to the application of the gate U_f which is implemented by choosing the operation time appropriately (see Table I). Instead of performing the second Hadamard gate the superposition is measured directly. This is possible because each superposition corresponds to a charge eigenstate: if f is constant, the charge state $|0\rangle$ is obtained while balanced f leads to charge state $|1\rangle$.

The additional advantage of this procedure is that the second sudden sweep brings the Hamiltonian back to a regime where the charge states are approximately the eigenstates such that the island charge is frozen. This is important since the time of the measurement is much bigger than the intrinsic time scale $\hbar/E_J$ of the qubit.

Thus, Nakamura's experiment with the operation times chosen according to Table I can be viewed as the implementation of the one-bit Deutsch algorithm. The only difference compared to the sequence of steps in Section II is that the single-qubit superpositions are prepared and measured directly, without performing additional one-qubit operations [28]. Of course, the essence of the algorithm is not affected: instead of rotating the state 'forward' by a Hadamard operation we apply a Hadamard rotation 'backwards' to the basis. It is easily seen that the coefficient $a_{|00\ldots0\rangle}$ of the charge state $|00\ldots0\rangle$ still obeys Eq. (3) where now the sum has to be taken over all $x \in \{+,-\}^N$.

The two-qubit Deutsch-Jozsa algorithm can be implemented by using two uncoupled qubits [24]. The gates $\sigma_z^{(1)} \otimes I_1^{(2)}$, $I_1^{(1)} \otimes \sigma_z^{(2)}$, $\sigma_z^{(1)} \otimes \sigma_z^{(2)}$ implementing the balanced functions (the upper index denotes the qubit number) and $I_1^{(1)} \otimes I_1^{(2)}$ for the constant function can be realized in complete analogy to the one-qubit algorithm. If, on measuring the qubits, both of them are found in the charge state $|0\rangle$ the function f was constant; if at least one qubit is found in the state $|1\rangle$, f was balanced.

Finally, it is also obvious that the Bernstein-Vazirani algorithm can be realized by applying the procedure described above. According to Eq. (6) one needs to implement the operation $(\sigma_z^{(j)})^{a_j}$ for the jth qubit which is straightforward by using the entries of Table I (note that the number b in Eq. (6) is irrelevant for the implementation). The measurement of the register then yields precisely the binary representation of a.

5 Implementation of algorithms involving entanglement – the three-qubit Deutsch-Jozsa algorithm

The realization of the three-qubit version of the algorithm is more difficult. Apart from the constant function 35 balanced functions need to be implemented.

Moreover, the three-qubit algorithm involves gates U_f which produce entangled final states.

The goal is to proceed along the same lines as above, that is, preparation of the state $|000\rangle$, sudden sweep of $n_x^{(j)}$ etc. The action of the gates U_f takes place in the basis $\{|+++\rangle, |++-\rangle, \ldots, |---\rangle\}$. That is, the gates operate at the degeneracy point $n_x^{(j)} = 1/2$ of the charge qubits. In order to find efficient ways for the implementation we first analyse the functions f and the corresponding gates U_f.

Apart from the constant function and its gate $I_1^{(1)} \otimes I_1^{(2)} \otimes I_1^{(3)}$ there are 7 balanced functions for which the gates are separable: $\sigma_z^{(1)} \otimes I_1^{(2)} \otimes I_1^{(3)}$, $I_1^{(1)} \otimes \sigma_z^{(2)} \otimes I_1^{(3)}$, $\ldots$, $\sigma_z^{(1)} \otimes \sigma_z^{(2)} \otimes \sigma_z^{(3)}$. Further there are 12 balanced functions for which the gates factorize into a one-qubit part and a two-qubit part as in example I) below. The other gates entangle all three qubits and can be divided into two classes (see example II) and III)). There are 12 gates of class II) and 4 gates of class III).

$$I)\quad \frac{1}{2}\left(I_1^{(1)} \otimes I_1^{(2)} + \sigma_z^{(1)} \otimes I_1^{(2)} - I_1^{(1)} \otimes \sigma_z^{(2)} + \sigma_z^{(1)} \otimes \sigma_z^{(2)}\right) \otimes \sigma_z^{(3)}$$

$$\begin{aligned} II)\quad \frac{1}{2}\Big(&\sigma_z^{(1)} \otimes I_1^{(2)} \otimes I_1^{(3)} - I_1^{(1)} \otimes I_1^{(2)} \otimes \sigma_z^{(3)} + \sigma_z^{(1)} \otimes \sigma_z^{(2)} \otimes I_1^{(3)} + \\ &+ I_1^{(1)} \otimes \sigma_z^{(2)} \otimes \sigma_z^{(3)}\Big) \end{aligned}$$

$$\begin{aligned} III)\quad \frac{1}{2}\Big(&\sigma_z^{(1)} \otimes I_1^{(2)} \otimes I_1^{(3)} - I_1^{(1)} \otimes \sigma_z^{(2)} \otimes I_1^{(3)} + I_1^{(1)} \otimes I_1^{(2)} \otimes \sigma_z^{(3)} + \\ &+ \sigma_z^{(1)} \otimes \sigma_z^{(2)} \otimes \sigma_z^{(3)}\Big) \end{aligned}$$

All separable single-qubit operations can be carried out in analogy with the one-qubit case above. In the following we discuss how the entangling gate operations can be achieved. For the realization of these gates a coupling of tunable strength between the qubits is required.

There are various ways to couple charge qubits [13, 14, 15, 17]. Here we investigate coupling via Josephson junctions [29]. Each qubit island is coupled to its nearest neighbour using a symmetric SQUID (see Fig. 3b).

Assuming that both the jth qubit and the jth coupling junction are tunable by local fluxes $\Phi^{(j)}$, $\Phi_K^{(j)}$ the Hamiltonian for the N-qubit system at the degeneracy point $n_x^{(j)} = 1/2$ reads

$$\begin{aligned} H_{N\mathrm{q}} &= \sum_{j=1}^{N} \Big\{ H_{1\mathrm{q}}^{(j)}(\Phi^{(j)}) \;+\; E_K^{(j)}\; \sigma_z^{(j)}\sigma_z^{(j+1)} \\ &\quad - (1/2)\; J_K^{(j)}(\Phi_K^{(j)})\; [\; \sigma_+^{(j)}\sigma_-^{(j+1)} \;+\; \mathrm{h.c.}\;] \Big\} \quad . \end{aligned} \tag{10}$$

Here $J_K^{(j)}$ is the Josephson energy of the jth coupling SQUID and $\sigma_\pm = (\sigma_x \pm$

$i\sigma_y)/2$. In the limit of small coupling capacitances $C_K^{(j)} \ll C^{(j)}$ we have

$$E_K^{(j)} = \frac{C_K^{(j)}}{2C^{(j)}} E_{\rm ch}^{(j)} \quad .$$

We will assume that $E_K^{(j)}$ is negligible. Since in practise the capacitive coupling is always present it is necessary to have $J_K^{(j)}(\Phi_K^{(j)} = 0) \gg 4E_K^{(j)}$. Then the dynamics of the system approximates the ideal dynamics sufficiently well.

Consider now the first and the second qubit coupled by $J_K^{(1)}$. By choosing, e.g. $-E_J^{(1)} = E_J^{(2)} = \pm J_K^{(1)} = \mathsf{J}$ and the operation time $t \simeq 0.97(2\pi/\mathsf{J})$ we obtain an operation similar to a swap gate for the qubits 1 and 2 for which we introduce the notation (in the basis $\{|++\rangle, \ldots, |--\rangle\}_{12}$)

$$[\pm 12] :=: \left(\begin{array}{cccc} 0 & 0 & 0 & \pm i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \pm i & 0 & 0 & 0 \end{array} \right) \quad . \tag{11}$$

By denoting one-bit phase shifts for the jth qubit

$$[\pm j] :=: \left(\begin{array}{cc} 1 & 0 \\ 0 & \pm i \end{array} \right) \quad , \tag{12}$$

we can write a sequence of operations which gives the two-bit entangling gate in example I) above:

$$[+1][-2] \text{---} [-12] \text{---} \sigma_z^{(3)} \quad . \tag{13}$$

After suddenly sweeping $n_x^{(1)}$ and $n_x^{(2)}$ to the degeneracy, first the one-bit phase shifts are performed by keeping $J_K^{(1)} = 0$. Then $J_K^{(1)}$ is switched on suddenly in order to do the two-bit rotation. The $\sigma_z^{(3)}$ rotation can be done at any moment since the third qubit is decoupled from the other two. Finally the $n_x^{(j)}$ are swept back suddenly and the register is measured. Note that the parameters of the one-bit and two-bit operations need to be chosen in a compatible way, i.e. the local Josephson couplings $E_J^{(1,2)}$ should be the same for the one-bit and two-bit operations in order to avoid unnecessary parameter switching.

There are numerous ways to represent the three-bit entangling gates. At least two different two-bit rotations need to be applied. During the second two-bit rotation the third qubit has to be 'halted'. This can be done by switching off both the E_J and the J_K which couple to this qubit. A possible sequence for example II) is

$$[+1][-2] \text{---} [+13] \text{---} [-12] \quad , \tag{14}$$

and for example III)

$$\sigma_z^{(1)} \sigma_z^{(2)} \sigma_z^{(3)} \text{---} [+12] \text{---} [+23] \text{---} [+12] \quad . \tag{15}$$

It is interesting to note that the completely entangling gates of class II) and III) can be realized approximately with a single three-qubit operation. In Table II and III we list the parameters for the various implementations including estimates for the accuracy of the respective operation. The complete set of entangling gates can be obtained from the sequences (13) – (15) by cyclic permutations of qubit numbers (and appropriate sign changes), thereby paying attention that the parameter settings are compatible for both one-bit and two-bit operations.

6 Conclusions

We have presented a possible implementation of the Deutsch-Jozsa algorithm in a setup of Josephson charge qubits. While this algorithm for a qubit number $N \geq 3$ requires entanglement we have demonstrated explicitely that the Bernstein-Vazirani algorithm does not involve entanglement for any number of qubits.

Our implementation realizes the algorithms by using only state-of-the-art technology. A peculiarity is that the gate operations representing the algorithm are carried out in a basis which is different from the one which is measured. This helps us to obtain the desired results with a minimum number of operations. Thus, one may hope to see the expected behaviour of the system even with the relatively small decoherence times which have been achieved up to now [18]. Of course, these measured decoherence times refer to a single qubit; at the moment it is not clear how much more difficult it is to observe entangled charge qubits experimentally. In fact, the experimental implementation of this proposal may serve to study this question in detail.

Finally, we mention that the methods outlined above can be used to study also other interesting problems such as the production and measurement of Bell states and GHZ states [30]. From a practical point of view, it would be particularly interesting to find ways to create such states in a 'single shot' with one appropriate gate operation. Even though it appears rather difficult to avoid the locality loophole in this kind of setup it is nevertheless a remarkable challenge to measure such quantum correlations in a macroscopic system.

7 Acknowledgements

The authors would like to thank L. Amico, D.V. Averin, I. Chuang, P. Delsing, G. Falci, J.B. Majer, Y. Makhlin, A. Osterloh, G.M. Palma, F. Plastina, C. Urbina, V. Vedral and C. v.d. Wal for stimulating discussions. This work was supported in part by the EC-TMR, IST-Squbit and INFM-PRA-SSQI.

References

[1] Barenco, A., Proc. R. Soc. London A **449**, 679 (1995);Deutsch, D., Barenco, A., and Eckert, A., Proc. R. Soc. London A **449**, 669 (1995); Weinfurter, H., Europhys. Lett. **25**. 559 (1994); Lloyd, S., Phys. Rev. Lett. **75**, 346 (1995); Barenco, A., Bennett, C.H., Cleve, R., DiVincenzo, D., Margolus, N., Shor, P.W., Sleator, T., Smolin, J., and Weinfurther, H., Phys. Rev. A **52**, 3457 (1995).

[2] Cory, D.G., Fahmy, A.F., and Havel, T.F., Proc. Nat. Acad. Sci. USA **94**, 1634 (1997).

[3] Gershenfeld, N.A. and Chuang, I.L., Science, **275**, 350 (1997).

[4] Chuang, I.L., Vandersypen, L.M.K., Zhou, X., Leung, D.W., and Lloyd, S., Nature **393**, 143 (1998).

[5] Jones, J.A., Mosca, M., and Hansen, R.H., Nature **393**, 344 (1998).

[6] Linden, N., Bariat, H., and Freeman, R., Chem. Phys. Lett. **296**, 61 (1998).

[7] Dorai, K., Arvind, and Kumar, A., Phys. Rev. A **61**, 42306 (2000).

[8] Arvind, Dorai, K., and Kumar, A., LANL e-print quant-ph/9909067 (1999).

[9] Collins, D., Kim, K.W., Holton, W.C., Sierzputowska-Gracz, H., and Stejskal, E.O., Phys. Rev. A **62**, 22304 (2000).

[10] Ahn, J., Weinacht, T.C., and Bucksbaum, P.H., Science **287**, 463 (2000).

[11] Kwiat, P.G.,Mitchell, J.R., Schwindt, P.D.D., and White, A.G., J. Mod. Optics **47**, 257 (2000)

[12] Leuenberger, M.N. and Loss, D., Nature **410**, p. 789 (2001).

[13] Shnirman, A., Schön, G., and Hermon, Z., Phys. Rev. Lett. **79**, 2371 (1997).

[14] Averin, D.A., Sol. State Comm. **105** 659 (1998).

[15] Makhlin, Y., Schön, G., and Shnirman, A., Nature **398**, 305 (1999).

[16] Mooij, J.E., Orlando, T.P., Levitov, L., Tian, L., van der Wal, C., and Lloyd, S., Science **285**, 1036 (1999).

[17] Falci, G., Fazio, R., Palma, G.M., Siewert, J., and Vedral, V., Nature **407**, 355 (2000).

[18] Nakamura, Y., Pashkin, Yu.A., Tsai, J.S., Nature **398**, 786 (1999).

[19] Friedman, J.R., Patel, V., Chen, W., Tolpygo, S.K., and Lukens, J.E., Nature **406**, 43 (2000).

[20] van der Wal, C., ter Haar, A.C.J., Wilhelm, F.K., Schouten, R.N., Harmans, C.J.P.M., Orlando, T.P., Lloyd, S., and Mooij, J.E., Science **290**, 773 (2000).

[21] Deutsch, D., Proc. R. Soc. London A **400**, 97 (1985).

[22] Deutsch, D. and Jozsa, R., Proc. R. Soc. London A **439**, 553 (1992).

[23] Cleve, R., Ekert, A., Macchiavello, C., and Mosca, M., Proc. R. Soc. London A **454**, 339 (1998).

[24] Collins, D., Kim, K.W., and Holton, W.C., Phys. Rev. A. **58**, R1633 (1998).

[25] Azuma, H., Bose, S., and Vedral, V., LANL e-print quant-ph/0102029 (2001).

[26] Bernstein, E. and Vazirani, U., in *Proc. 25th Ann. ACM Symp. on the Theory of Computing.* New York, ACM Press 1993.

[27] Meyer, D.A., LANL e-prints quant-ph/0004092, quant-ph/0007070 (2000).

[28] In Section 2, the Hadamard transformations of the steps *(ii)* and *(iv)* are present only for 'technical reasons': it is assumed that the basis states on which the gate U_f acts (the computational basis) are the same states which can easily be prepared and measured.

[29] Siewert, J. Fazio, R., Palma, G.M., and Sciacca, E., J. Low Temp. Phys. **118**, 795 (2000).

[30] Plastina, F., Fazio, R., and Palma, G.M., LANL e-print cond-mat/0105029 (2001).

f	gate U_f	time t
constant	I_1	$2\pi\hbar/E_J$
balanced	σ_z	$\pi\hbar/E_J$

Table 1:

gate	implementation	$E_J^{(1)}$	$E_J^{(2)}$	$E_J^{(3)}$	$J_K^{(1)}$	$J_K^{(2)}$	$J_K^{(3)}$
$II)$	sequence (9)	-J	J	J	-J	0	J
$II)$	single operation	-J/2	0	J/2	J	J	0
$III)$	sequence (10)	J	-J	J	J	J	0
$III)$	single operation	J/2	-J/2	0.83J	0	J	J

Table 2: Junction parameters for various realizations of the gates $II)$ and $III)$.

gate	implementation	time $t/(2\pi\hbar/\mathsf{J})$	$a_{\|0\ldots0\rangle}$ $(E_K^{(j)}=0)$	$a_{\|0\ldots0\rangle}$ $(E_K^{(j)}=\mathsf{J}/40)$
$II)$	sequence (9)	0.97 (2bit op.)	$2\cdot10^{-3}$	$2\cdot10^{-4}$
$II)$	single operation	0.80	$7\cdot10^{-5}$	$2\cdot10^{-2}$
$III)$	sequence (10)	0.97 (2bit op.)	$3\cdot10^{-4}$	$6\cdot10^{-3}$
$III)$	single operation	1.19	$<10^{-5}$	$2\cdot10^{-3}$

Table 3: Gate operation times and accuracy for gate realizations of examples $II)$ and $III)$. The coefficient $a_{|0\ldots0\rangle}$ is a measure for the fidelity of the operation (for an ideal operation $a_{|0\ldots0\rangle}=0$, cf. Eq. (3)). The operation time for the sequences refers to the time needed for the two-qubit rotations. Single-qubit rotations are assumed to be perfect.

Figure 1: The sequence of operations to perform the Deutsch algorithm on a register of N qubits. According to Ref. [23] it can be interpreted in terms of quantum interferometry. The first Hadamard transformation produces a superposition of <u>all</u> possible states. Thus, with the application of the f-controlled gate U_f the outcome of f for all possible arguments is evaluated at the same time. The second Hadamard transformation brings all computational paths together.

Figure 2: The qubit is prepared in the ground state $|0\rangle$. After suddenly sweeping the gate voltage the system starts Rabi oscillations between the eigenstates of the new Hamiltonian $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$. After the time t the gate voltage is swept back suddenly which freezes the final state; then the qubit is measured.

Figure 3: a) A charge qubit. The Josephson energy of the junction can be controlled by the magnetic flux Φ: $E_J(\Phi) = 2\mathcal{E}_J \cos(\pi\Phi/\Phi_0)$, where $\mathcal{E}_J$ is the Josephson energy of the junctions of the symmetric SQUID and $\Phi_0 = h/(2e)$ is the flux quantum [15]. b) A possible realization of coupled charge qubits.